algebra by a consecutive formulation of Kronecker's abstractions without using the notion of a limit, and restricts himself in the process to those parts of the group-theory that are necessary for his purpose ; Vogt provides for French readers an equivalent of what Netto, in his older book, and Netto's translators, Battaglini and Cole, had given to Germans, French, and English ; in Weber's great treatise the theory appears as an important aspect, but still only an aspect, of the problem of algebra. For all these books the theory of groups is a means to an end, or rather to one of multifarious ends which it can be called upon to serve, viz. the algebraic solution of equations.

But in course of time the theory of groups has ever more and more emancipated itself from algebra, whose servant it originally was. Years ago Cayley threw out, though he did not develop, the fundamental idea that the notion of a group, in and for itself, is in no way bound up with permutations and substitutions, but arises whenever the effect of two operations performed successively upon an object is the same as that of a single operation of the same kind. In the general theory of groups thus founded, groups of finite order form a well-marked division. To bring together the great mass of single results yielded by exploring this division of the theory from different sides, to take a comprehensive view of them, and to exhibit them in a well-digested form, is the problem that has been attempted for the first time in the book before us, and it is solved in the happiest manner.

The author does not push abstraction to the point of banishing from his book all concrete methods of representation of groups. He frequently uses properties of groups of substitutions, in particular, not merely with a view to making results plain to intuition, but also for the deduction from them of properties of groups in the abstract. He himself asks the question why other particular methods of representation of a group, *e.g.* by means of homogeneous linear transformations, are not employed in a similar way, and he answers it, as I think rightly, in the words "that, while in the present state of our knowledge, many results in the pure theory are arrived at most readily by dealing with properties of substitution groups, it would be difficult to find a result that could be most directly obtained by the consideration of groups of linear transformations."

Believing that familiarity with symbolical calculations concerning interchanges of letters is not to be assumed on the part of his countrymen, the author gives a sketch of this theory in a short first chapter. The second chapter begins with Cayley's abstract definition, cited above, and carries the development of the general properties of a group as far as Dyck's theorem to the effect that every group of finite order N can be represented as a group of interchanges of N symbols. The third chapter develops the notions : sub-group, self-conjugate sub-group, simple group, isomorphism, factor groups. Then follow in chapters four and five special investigations relating to Abelian groups and to groups whose order is a prime number ; in the latter the author has placed a series of results of his own researches. The sixth chapter brings us back with Sylow's theorem to the general theory, and the pivot of the theory is found, in chapter seven, in the theorems on the composition-series of a group. The three following chapters are especially concerned with groups of substitutions, with the questions of their transitivity and primitivity ; in addition to the general theorems, they contain a great number of completed researches on special groups and types of groups ; some of these are of intrinsic interest, and others serve as vivid illustrations of the general theory. The eleventh chapter treats of the isomorphism of a group with itself on the lines followed by Hölder and Frobenius. In the twelfth and thirteenth chapters is explained the method of representation of a group which was developed in a general manner by Dyck, viz. the representation by means of geometrical transformations of a surface divided into regions, and especially by the transformations effected by linear relations between complex variables. An older method, due to Cayley, of representation by coloured diagrams leads in simple cases to results easier to appreciate at a glance ; this method also is here expounded and illustrated by a beautifully executed coloured plate. The fourteenth chapter treats of the representation of a group by means of systems of linear congruences. There is a certain incongruity in the fact that while this is inserted, the representation by means of systems of linear equations is omitted ; but the theory of this method would by itself furnish material for a second book as large as the one before us. Perhaps the author will some day give us such a book. On the other hand we find here the extension of the theory named to congruences holding among Galois' imaginaries. This was cultivated in his time by E. Mathieu, and has been taken up recently by E. H. Moore and the author of this treatise, in whose hands it has led, among other remarkable results, to the knowledge of a new series of types of finite groups. Finally the last chapter gives an account of the most modern enumerations of types of groups, in particular of simple groups of an order which can be expressed in a prescribed manner as a product of primes.

The author has not attempted to give historical references concerning the discovery of the older theorems ; but for the more modern literature, of about the last twenty years, and right up to the time of publication, his references are full and trustworthy.

In respect of the completeness and exactness of its matter, the work fills in a very acceptable manner a gap in the literature of mathematics, and not merely of mathematics in England. It brings to the special subject a great wealth of material for the widening and deepening of knowledge ; while at the same time beginners, under expert guidance, will be able to make a profitable use of it.

H. BURKHARDT.

OUR BOOK SHELF.

Radiation : an Elementary Treatise on Electro-magnetic Radiation, and on Röntgen and Cathode Rays. By H. H. Francis Hyndman, B.Sc. (Lond.). With a Preface by Prof. Silvanus P. Thompson, D.Sc., F.R.S. Pp. xviii + 307. (London : Swan Sonnenschein and Co., Ltd. New York : The Macmillan Co., 1898.)

THE author considers chiefly those portions of the subject which are somewhat neglected in most books, and in addition deals with the results of some of the

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more recent investigations. Brief outlines are, however, given of those parts that are not especially treated, in order to maintain continuity and allow of a direct comparison of the properties of the different radiations. Many references are given to original sources, which enable the reader to extend more readily the narrow limits of the book.

The treatise contains an account of a large amount of interesting work, but it can scarcely be expected that one not perfectly familiar with the subject can follow the ideas so briefly expressed in many places.

Throughout the volume we are confronted with a considerable number of misleading and erroneous statements, the nature of which can be seen from a few examples. For instance, on p. 255 it is stated :—"That the rays do not themselves act as conductors seems proved, for they can pass between bodies of very different potentials if these are not already connected by lines of force." Any comment on this is superfluous.

On p. 198 we read :—"The cathode rays possess the same power as the ultra-violet and the X-rays, of causing those gases which they traverse to become conductors of electricity." All careful experiments, however, show that ultra-violet light in traversing a gas does not produce any conductivity.

Again, on p. 259 it is stated that the charge carried by the ions in X-ray conduction can be calculated from the values of the saturation current and the time required for the conductivity of the gas to sink to half of its value. Such a calculation is not possible.

On p. 254, in regard to conduction produced by the passage of X-rays through bodies, we read that "paraffin, wax, sulphur, ebonite, mica, and all gases become conductors." It is now generally believed, however, that solid insulators are not made to conduct by the rays, and the author ignores this side of the question.

Strict accuracy in the facts presented is an essential of every scientific treatise, as the harm produced by the inception of erroneous ideas cannot be over-estimated.

J. Z.

Four-footed Americans and their Kin. By Mabel Osgood Wright. Pp. xv + 432 ; illustrated. (London : Macmillan and Co., Ltd. New York : The Macmillan Company, 1898.)

WHATEVER difference of opinion may obtain as to whether the title of this little volume is the best that could have been found, or whether the "Mr. Barlow" style of "Sanford and Merton" is the most suitable for popular natural history, there can be no hesitation in according the highest praise to the exquisite illustrations which form its most attractive feature. In addition to being in every way artistic productions these, so far as we can judge, are absolutely life-like ; portraying the animals not only among their natural surroundings, but also in characteristic attitudes. Where none are bad, and all attain a high standard of excellence, it is almost invidious to make a selection ; but two of the illustrations which especially strike our fancy are the wood-hare on p. 141, and the little striped skunk on p. 181. It is rare, indeed, to meet with illustrations of such a high class, and these alone ought to give the book a large sale ; while the artist, Mr. E. S. Thompson, ought to obtain a world-wide reputation as a delineator of animal life.

But it is not on the illustrations alone that the book has to depend for popularity ; as, allowing for certain peculiarities in the style, the descriptions of the habits of the various animals mentioned are for the most part well written and entertaining. A classified synopsis at the end gives a clue to the identification of the species described ; and in this connection it is important to notice that in nearly all cases the author is thoroughly up to date as regards nomenclature. As changes in nomenclature have been necessary, it is most desirable that they should be adopted by popular writers. We must, however, take leave to object to the title of "Beef or Meat Family" for the *Bovidae*, as also of "Antelope Family" for the Prongbuck, and of "Leapers" for the *Leporidae*. Since, too, the *Viverridae* are conspicuous by their absence in America, the misnomer of "Civet-cat" is decidedly objectionable for the Cacomistle (*Bassariscus*), whose inclusion among the "Cousins of Cats" is scarcely warranted by the facts. As a Christmas present to all young persons interested in animals, the work may be confidently recommended.

R. L.

Electricity made Easy, by Simple Language and Copious Illustration. By Edwin J. Houston, Ph.D., and Arthur E. Kennelly, Sc.D. Pp. 348. (London : Swan Sonnenschein and Co., Ltd., 1898.)

SIMPLE descriptions and explanations of electrical appliances met in every-day life are given in this volume. Facts with regard to the nature and functions of electric mains, the distribution of currents, generating stations, the principles of incandescent and arc lighting, measurement of electric supply, electric bells, telegraphy, telephones, and other things concerned in the production and uses of electricity are stated in the simplest of words. Illustrations are given of the external and internal appearance of many of the appliances described. These pictures are, perhaps, a trifle too numerous, for they tend to give the volume the character of a catalogue of electric fittings.

The comparison of the flow of water with the flow of electricity is a useful one ; but there is a danger that the reader will carry the analogy too far. On p. 30, for instance, we read that electricity "flows through wires very much like water or gas flows through pipes, but flows through a solid wire as readily as through a tubular wire of the same weight." Brief statements of this kind are frequently the cause of serious misconceptions as to the nature of electricity.

Differential and Integral Calculus for Technical Schools and Colleges. By P. A. Lambert, M.A., Assistant-Professor of Mathematics, Lehigh University. Pp. x + 245. (New York : The Macmillan Company. London : Macmillan and Co., Ltd., 1898.)

THE first hundred pages of this book are occupied with the differentiation and integration of simple algebraic functions, including easy applications to geometry, and here and there a mechanical problem is set amongst the examples.

Then come chapters involving trigonometrical functions, and logarithmic and exponential functions, followed by a chapter containing applications to mass centres and moments of inertia. The next two chapters deal with expansions and applications of Taylor's theorem, and the last forty pages of the work are devoted to an outline discussion of certain simple classes of differential equations.

There are some good features in the book considered as an introductory text-book for ordinary school use where a good deal of supplementary oral teaching would be given. Thus the statement of methods and the proofs, or indications of proof, are clearly worded ; the examples are easy and straightforward, and in two or three cases distinctly fresh illustrations appear.

But the general treatment is very superficial ; far too much ground is covered ; there is an appearance of easy progress produced by the avoiding of the real difficulties which occur in the practical applications of the calculus. In fact, it is quite misleading to describe the work as specially adapted for technical schools and colleges. No instructor of an English technical class would consider this handbook an adequate introduction to the application of the calculus to physics or engineering.

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